American Monte Carlo for Bermudan CVA

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Outline

Background

Problem

Way Out

Example and Results

Appendix

Quaternion Risk Management

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- Problem
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Background

Software and services around pricing, market and credit risk analytics

Quaternion Risk Engine (QRE) based on QuantLib

CVA/DVA and PFE:

- Netting and collateral
- Unilateral/bilateral risk
- Cross asset - IR, FX, INF, EQ, COM, CR

QRE

CVA processes after data loading:

- **Market scenario generation**
  Needs cross asset risk factor evolution models, free of arbitrage
- **NPV cube generation**
  Needs fast pricing and parallel processing
- **Post processing**
  Needs efficient large data handling for "cube" analysis, aggregation of netting sets, collateral modelling, expected exposure and ultimately CVA/DVA calculation

QRE

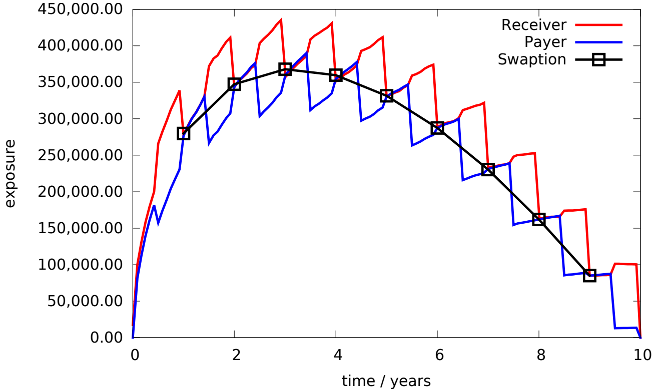
How we do it...

- Simulated scenarios populate **QuantLib quotes** which are linked to **QuantLib term structures** (we make sure that observer chains are not overloaded)
- Update Settings::instance().evaluationDate() as we move forward through time
- Update fixing history on the path as we move forward
- Reprice the portfolio with engines linked to the term structures above

The portfolio does not "know" that it is priced on a Monte Carlo scenario rather than a "real" market data set: We can use instruments and engines in QuantLib, as well as additional ones.

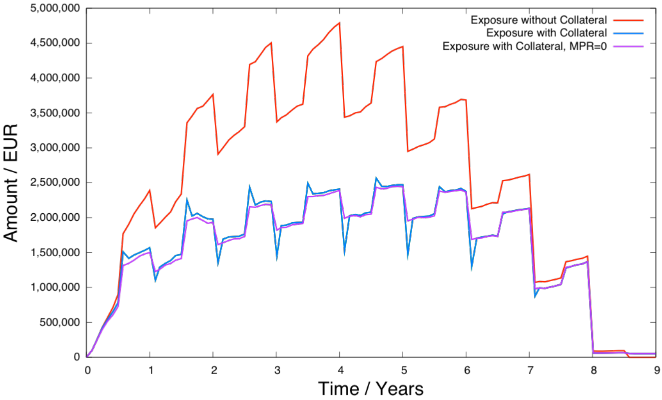
Single Ccy Swap Exposure

ATM Single Currency Vanilla Swap, A fixed vs. S floating



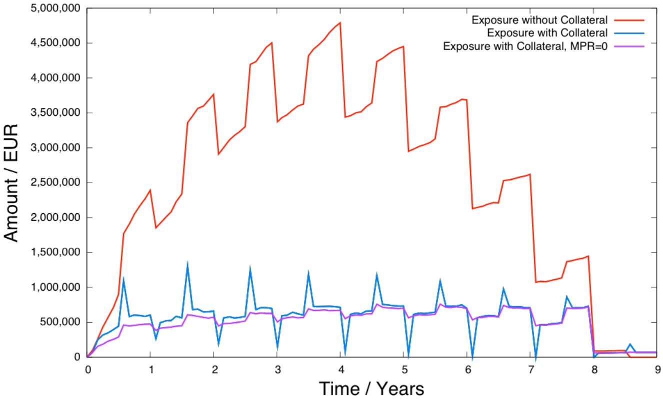
Single Ccy Swap Exposure with Collateral

Threshold 4m EUR, MTA 0.5m EUR, MPR 2 Weeks



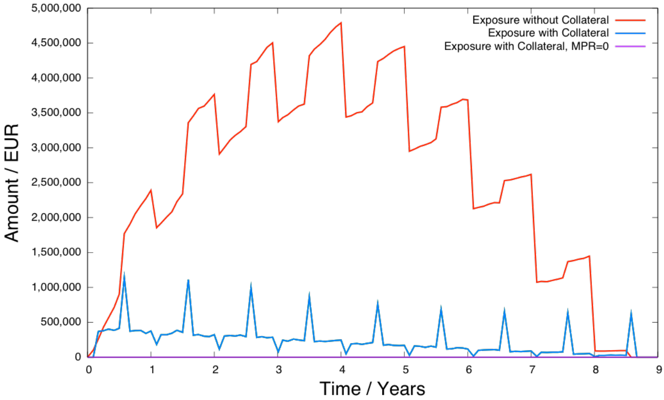
Single Ccy Swap Exposure with Collateral

Threshold 1m EUR, MTA 0.5m EUR, MPR 2 Weeks



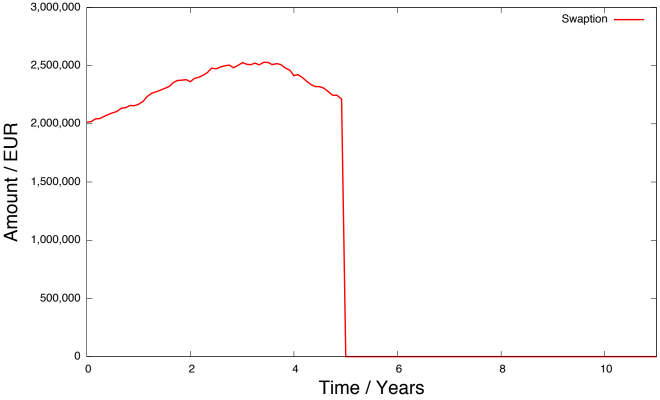
Single Ccy Swap Exposure with Collateral

Zero threshold, MPR 2 Weeks



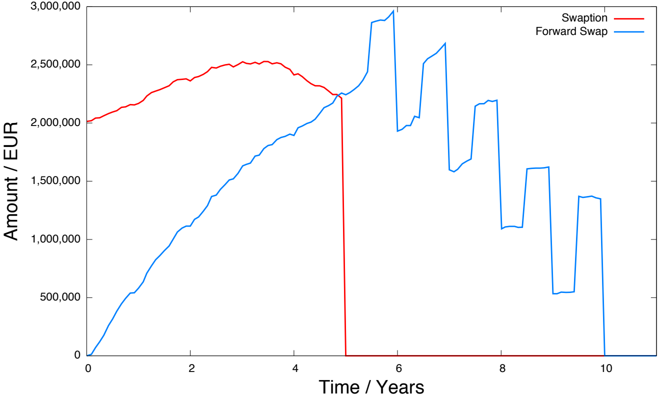
European Swaption Exposure

European Swaption Exposure, Expiry 5Y, Cash Settlement



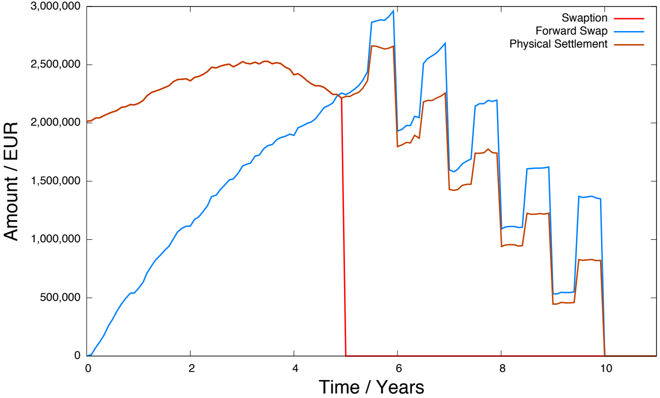
European Swaption Exposure

Underlying Swap, Forward Start in 5Y, Term 5Y

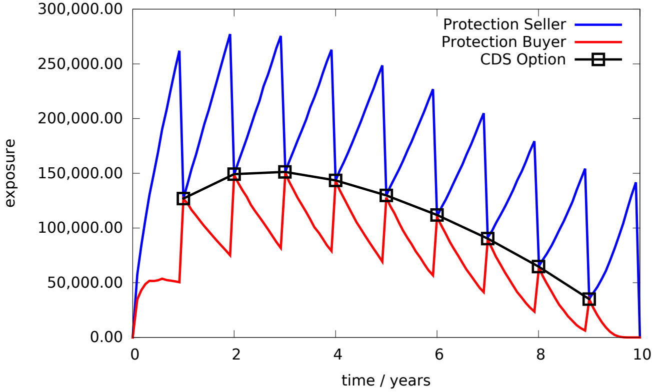


European Swaption Exposure

European Swaption with Physical Settlement



CDS Exposure

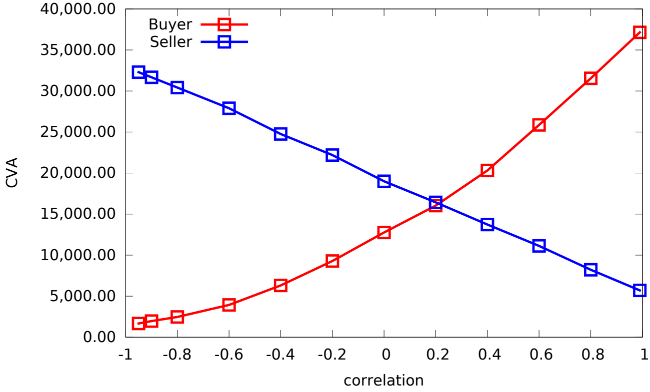


Quaternion Risk Management

CDS and Wrong Way Risk

Varying the correlation between hazard rate processes of ref. entity and counterparty

CDS: 10m EUR notional, 10Y term, ATM



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Bermudan Exercise

How to - naively - handle a Bermudan Swaption in this framework?

Like vanilla trades - we price the swaption

- under each scenario ($\sim$ 10000)
- and for each future point in time ($\sim$ 120 with monthly steps out to 10y for collateral tracking)
- i.e. **about a million times**

Bermudan Exercise

So how long does that take without parallelization?

About **3 milli sec** per price on our LGM grid (without re-calibration), i.e. about **50 min** in total.

Compare that to a vanilla swap with about **30 micro sec** per price or **0.5 min** in total.

This can be a problem when the portfolio has a significant number of multi-callables.

Quaternion Risk Management

Outline

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American Monte Carlo

American Monte Carlo (published 2001 by Longstaff and Schwartz) is a method that allows pricing of American/Bermudan exercise features in a Monte Carlo setting.

The expected continuation values - for making exercise decisions on each path - are estimated by regression analysis across the Monte Carlo scenarios. See the original LS example in the appendix.

There are implementations of the LS algorithm in QuantLib, see e.g.

- **Klaus Spanderen's American Equity Option**
- **Mark Joshi's Market Model**.

American Monte Carlo

Why is this promising from a CVA perspective?

- The LS algorithm produces NPVs of the underlying instrument and the option along each path on exercise dates
- The swaption exposure profiles for CVA can be extracted as a swaption pricing by-product
- One can handle both cash and physical exercise in the algorithm
- The exposure evaluation can be extended to interim grid points
- We can re-use the Monte Carlo market scenarios generated for the "outer" CVA loop

American Monte Carlo

LS algorithm in a nutshell

- generate market scenarios (trigger paths),
  price the underlying (Swap) along each path
- perform one rollback with regressions on each exercise date
- generate market scenarios again (valuation paths),
  price the underlying again along each path

At first glance, this should make the CVA analysis for a Bermudan swaption only 2-3 times more expensive than for the underlying, and about 50 times faster than with brute force evaluation of Bermudan swaptions under scenarios on all grid dates.

Let us check ...

Outline

Example and Results

Extreme Bermudan Swaption example:

- Swap Maturity: 30/09/2039
- Exercise: Annual between 30/09/2019 and 30/09/2038
- Notional: 100,000,000 EUR
- Pay: 3% annual 30/360
- Receive: 6m-Euribor semi-annually

Example and Results - Cash Settlement

Hagan LGM grid

- **NPV: 10.634** Mio EUR
- **Time: 17.5 ms** (quick, but longer than in our estimate above)
- Grid: $s_y = 4.0$, $n_y = 10$, $s_x = 4.0$, $n_x = 18$
  (minimum parameter values recommended by Hagan)

AMC pricing

- **NPV: 10.636** Mio EUR
- Forward time: **1031 ms** (path generation and underlying pricing)
- Rollback time: **62 ms** (regressions)
- Forward time: **641 ms** (underlying pricing **until exercise**)
- Samples: 10000
- Time steps: 300 (monthly rather than annually on exercise dates)

Example and Results - Cash Settlement

AMC and grid prices are surprisingly close (0.02 % price difference)

AMC pricing is slow, **about 2 sec** vs about **20 milli sec** on the grid

... but it generates in **2 sec** the swaption exposure profile for CVA with high resolution (10,000 samples, monthly time steps) which would take about **50 min** with brute force Bermudan pricing under scenarios, according to our rough estimate.

Where does this large difference come from?

1. We evaluate only swaps through the paths/scenarios, which costs less than evaluating Bermudan swaptions as in the crude method
2. We evaluate the underlying swap on 20 exercise dates only (for cash settlement!), even if we need to produce exposures on 300 dates or more in between.

Example and Results - Physical Settlement

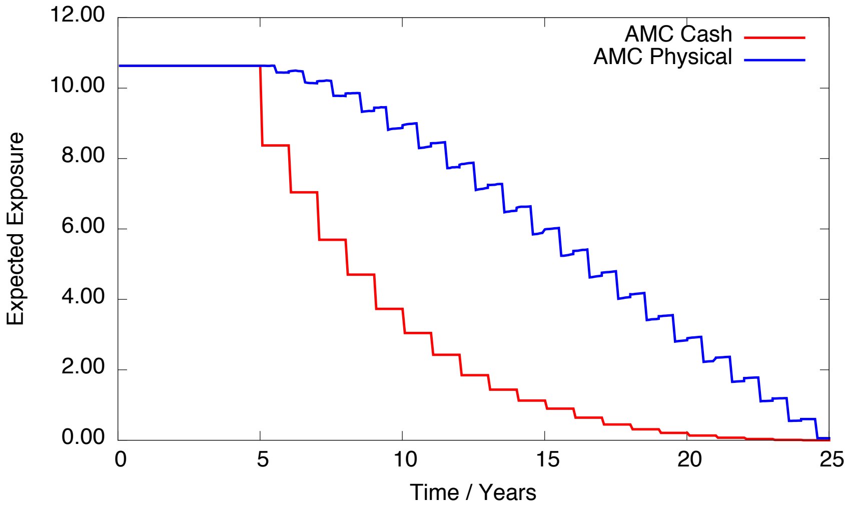
AMC pricing for **physical exercise**

- **NPV: 10.636** Mio EUR
- Forward time: **1019 ms** (path generation and underlying pricing)
- Rollback time: **61 ms** (regressions)
- Forward time: **6025 ms** (underlying pricing)
- Samples: 10000
- Time steps: 300 (monthly rather than annually on exercise dates)

Why has the second "forward time" gone up to 6 sec?

- Physical: Evaluate the underlying on each grid point after expiry through to final maturity.
- Cash: Zero exposure contributions after exercise instead

Example and Results - Exposure Profiles



Thank you

Outline

Longstaff-Schwartz Example (1)

American equity put option with strike price $K = 1.10$ and expiry at t_3.
Stock prices X_i, exercise values $E_i = (K - X_i)^+$:

Path	X_0	X_1	E_1	X_2	E_2	X_3	E_3
1	1.00	1.09	0.01	1.08	0.02	1.34	-
2	1.00	1.16	-	1.26	-	1.54	-
3	1.00	1.22	-	1.07	0.03	1.03	0.07
4	1.00	0.93	0.17	0.97	0.13	0.92	0.18
5	1.00	1.11	-	1.56	-	1.52	-
6	1.00	0.76	0.34	0.77	0.33	0.90	0.20
7	1.00	0.92	0.18	0.84	0.26	1.01	0.09
8	1.00	0.88	0.22	1.22	-	1.34	-

Longstaff-Schwartz Example (2)

Regression at time 2:

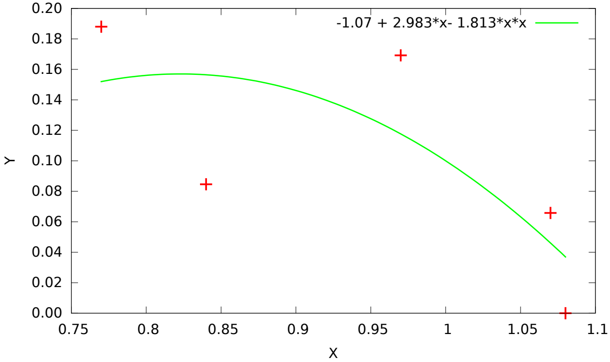
- Y_2: Payoff at t_3 discounted back to t_2
- $C_2 = \mathbb{E}[Y_2|X_2]$: Continuation value at t_2
- Exercise at t_2 if $E_2 > C_2$

Path	E_2	C_2	Y_2	Exercise at t_2 ?
1	0.02	0.0369	0.94×0.00	
2	-	-	-	
3	0.03	0.0461	0.94×0.07	
4	0.13	0.1176	0.94×0.18	Y
5	-	-	-	
6	0.33	0.1520	0.94×0.20	Y
7	0.26	0.1565	0.94×0.09	Y
8	-	-	-	

Longstaff-Schwartz Example (3)

The continuation value at t_2 is estimated by regression across paths that are in the money at t_2:

$$C = \mathbb{E}[Y|X] = f(X) = -1.07 + 2.983\,X - 1.813\,X^2$$



The regression fits $f(X)$ by minimising $\sum_i (Y_i - f(X_i))^2$; it essentially averages over continuation values Y with similar associated exercise values X, bundling paths passing through the neighbourhood of X.

Quaternion Risk Management

Longstaff-Schwartz Example (4)

Regression at time 1:

- Y_1: Payoff at t_2 or t_3 discounted back to t_1
- C_2: Continuation value $C_1 = \mathbb{E}[Y_1|X_1] = f(X_1)$ by regression across paths that are in the money at t_1, i.e. $E_1 > 0$
  $\Rightarrow f(X) = 2.038 - 3.335\,X + 1.356\,X^2$

Path	E_1	C_1	Y_1	Exercise at t_1 ?
1	0.01	0.0139	$0.94\times$ 0.00	
2	-	-	-	
3	-	-	-	
4	0.17	0.1092	$0.94\times$ 0.13	Y
5	-	-	-	
6	0.34	0.2866	$0.94\times$ 0.33	Y
7	0.18	0.1175	$0.94\times$ 0.26	Y
8	0.22	0.1533	$0.94\times$ 0.00	Y

Longstaff-Schwartz Example (5)

Exercise summary:

Path	Exercise at t_1	Exercise at t_2	Exercise at t_3
1			
2			
3			Y
4	Y	Y	Y
5			
6	Y	Y	Y
7	Y	Y	Y
8	Y		

Pricing: Discount payoff from earliest exercise and average over paths.

Least Squares Monte Carlo (LSM) Algorithm

1. Compute exercise values E_{ij} for all paths i and exercise dates j
2. Roll back from exercise t_{n+1} to t_n
   - Discount the path payoffs to t_n from the next exercise value where the exercise decision was positive: Y_{in}
   - Regression analysis across all (X_{in}, Y_{in}) where $E_{in} > 0$ to find the parameters a, b, c in $\mathbb{E}(Y_n|X_n) = f(X) = a + b\,X + c\,X^2$
   - Compute continuation values for all paths i, $C_{in} = f(X_{in})$
   - Exercise decision for all paths i: Positive if $E_{in} > C_{in}$
3. Pricing: Discount payoffs from earliest exercise (where decision was positive); average over all paths

Quaternion Risk Management

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