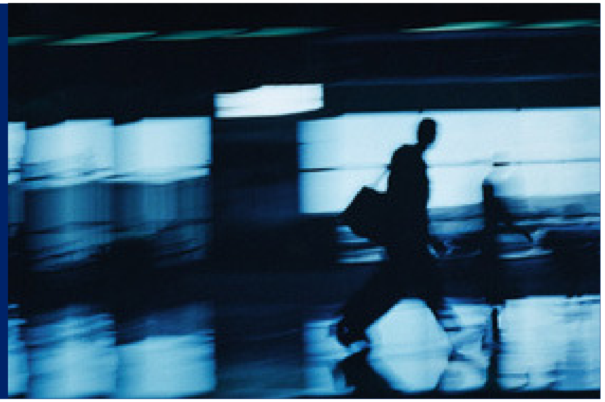
Pricing of Accreting Swaptions using QuantLib

Dr. André Miemiec, 13./14. Nov. 2013



Introduction

- Origin of the problem:
  - Valuation of Multicallable Accreting Swaptions
- Elementary Observations:

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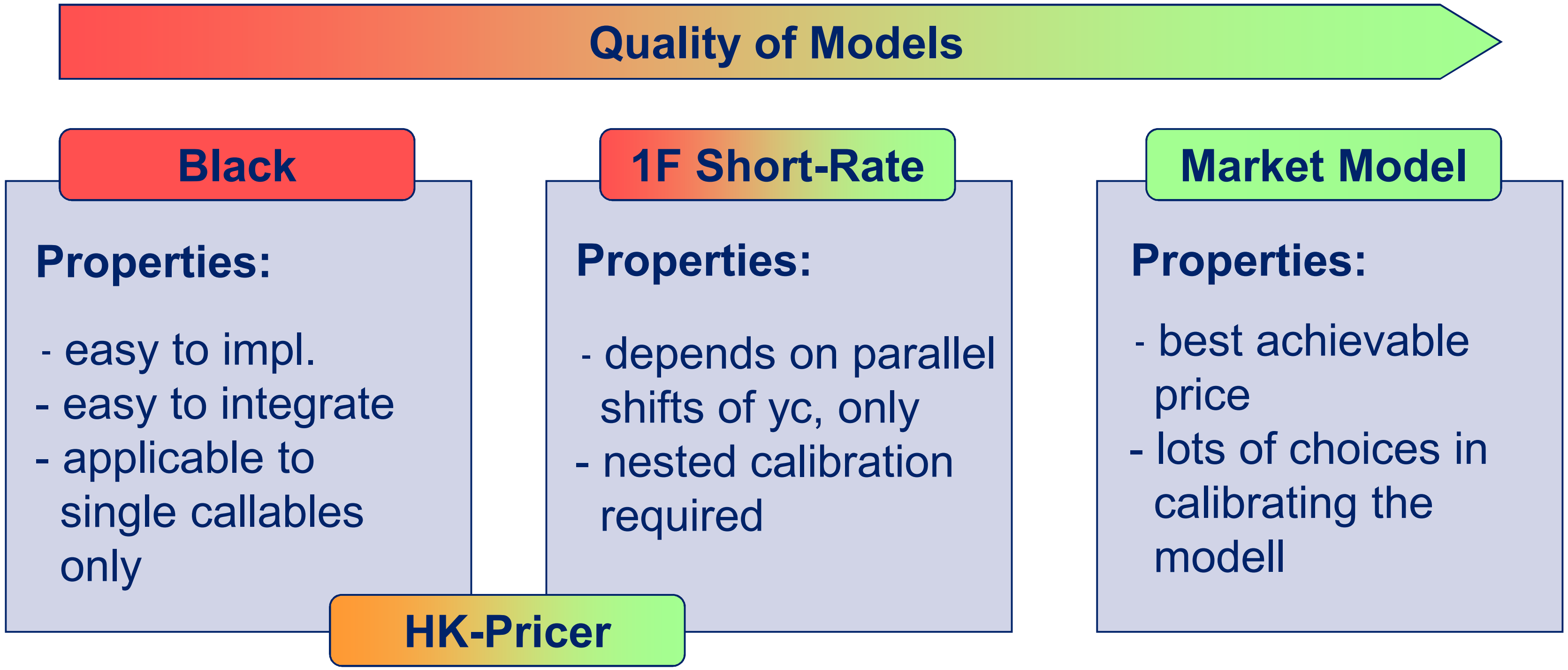
- Reason must be traced back to the model choice or calibration, respectively

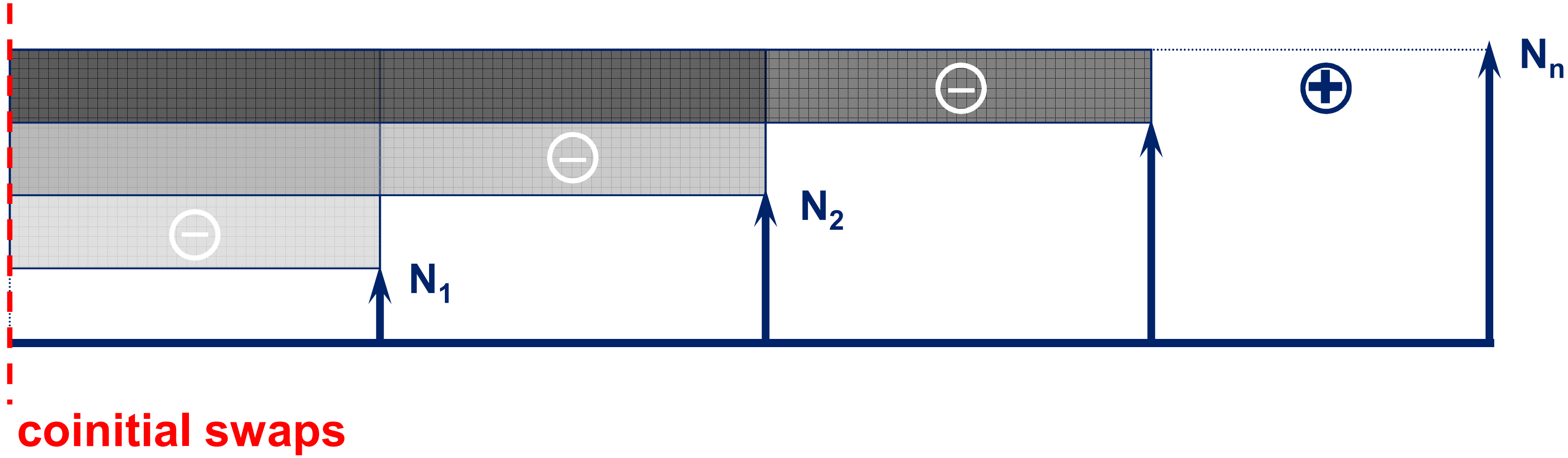


- Amortising swaptions are most sensitive to ‘parallel’ moves in the yield curve, so a single factor model is sufficient ⇨ LGM

- **QuantLib** does not provide a LGM implementation but posseses an unsatisfactory implementation of Hull-White
- Calibration Issue:
  - Accreters are calibrated to coinitial not coterminal swaptions
  - HW is unable to cope with this requirement.
- I had to decide between two alternatives:
  - do a proper LGM implementation or
  - do the calibration otherwise.
- Made the second choice because
  - the method selected combines the best properties of the Black and 1F-Short-Rate Models.

1. Introduction
2. Model Description
3. Implementation in QuantLib
4. Pricing Quality

- Irregular swap and its decomposition into a basket of regular swaps



- Hagan*: Want to exercise all basket swaps at the same time, i.e. put them equally far (λ) from ATM (K_i)

$$R_i = K_i + \lambda$$

*The corresponding reference can be found at the end of the talk.

- Bond model of a accreting swaption:

Fixed Leg:
$$C_i = N_i \tau_i K + N_i - N_{i-1}$$
$$C_n = N_n \tau_n K + N_n$$

Float Leg:
$$\tilde{C}_1 = N_1$$

- Basket of standard swaps with par-rates $\{K_i\}_{i=1..n}$ and notionals $\{A_i\}_{i=1..n}$

$$\begin{pmatrix} 1+(K_1+\lambda)\cdot\tau & (K_2+\lambda)\cdot\tau & \cdots \\ 0 & 1+(K_2+\lambda)\cdot\tau & \\ \vdots & & \ddots \end{pmatrix} \cdot \begin{pmatrix} A_1 \\ A_2 \\ \vdots \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \\ \vdots \end{pmatrix}$$

- Matching the floating leg:

$$N_1 = \sum_{i=1}^{N} A_i(\lambda) \quad \Rightarrow \quad \lambda$$

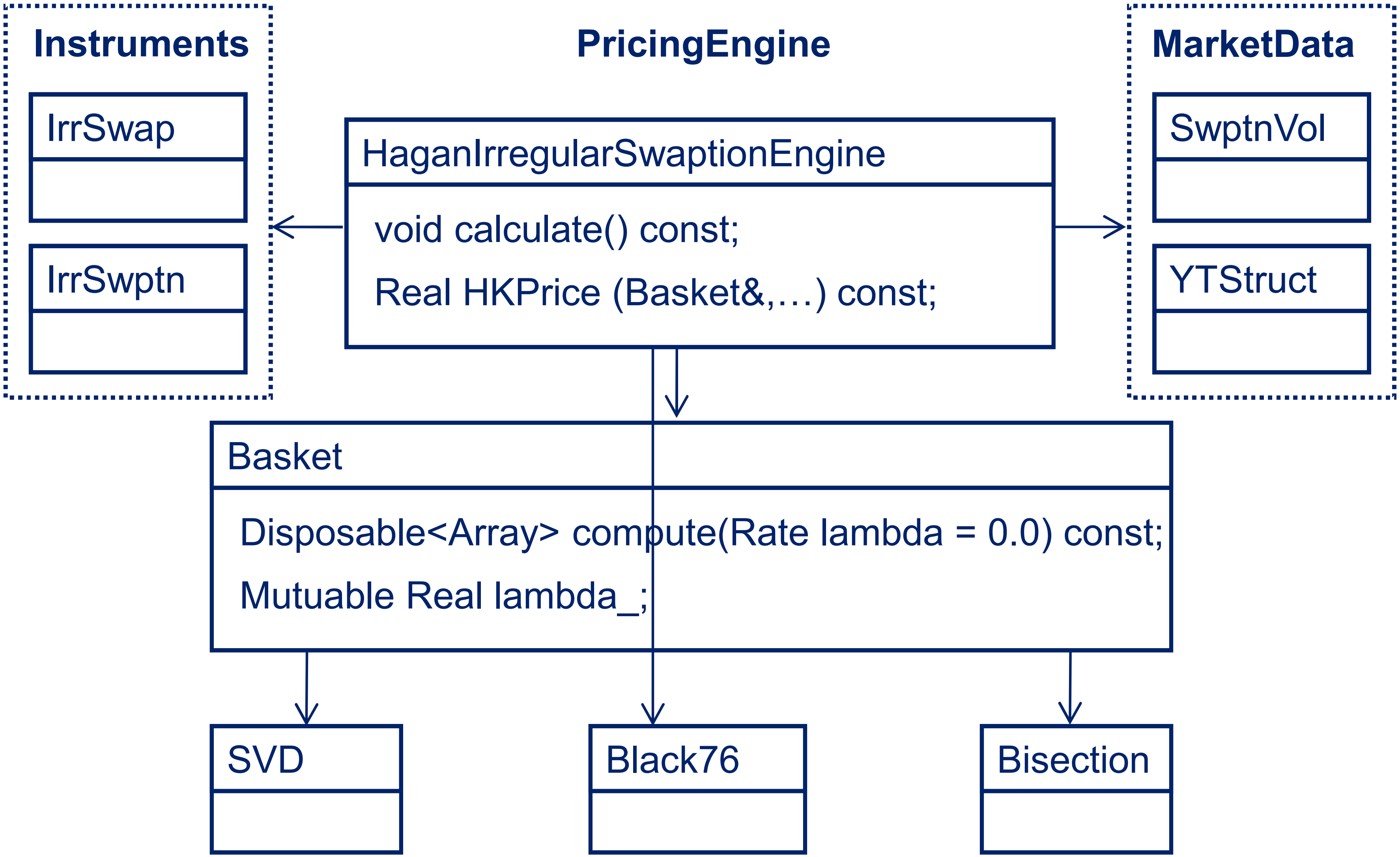
- Basket decomposition: $$U_t = \sum_{i=1}^{N} A_i\, U_t^i(R_i)$$
- Hunt-Kennedy**:
  - Select r^* such that: $$U_t(r^*) = 0$$
  - Select R_i such that: $$U_t^i(R_i, r^*) = 0$$ ↯
- Then

$$V_t(r) = e^{-rt} \cdot E\left[U_t^+(r) \right] = \sum_{i=1}^{N} V_t^i(R_i, r)$$

- This decomposition works pretty well, if Hagan's R_i are actually used.
  - Typical deviation to a properly calibrated LGM modell some $100

The corresponding reference can be found at the end of the talk.

- Basic structure of the algorithm



- Final Pricing Function:

```
Real  HKPrice(Basket& basket,boost::shared_ptr<Exercise>& exercise) const {

  boost::shared_ptr<PricingEngine> blackSwaptionEngine = boost::shared_ptr<PricingEngine>(
                              new   BlackSwaptionEngine(termStructure_,volatilityStructure_));

  Disposable<Array> weights = basket.weights();

  Real npv = 0.0;

  for(Size i=0; i<weights.size(); ++i){
     boost::shared_ptr<VanillaSwap> pvSwap_ = basket.component(i);
     Swaption swaption = Swaption(pvSwap_,exercise);
     swaption.setPricingEngine(blackSwaptionEngine);
     npv += weights[i]*swaption.NPV();
  }
  return npv;
}
```

Side remark on standard QL-Classes:

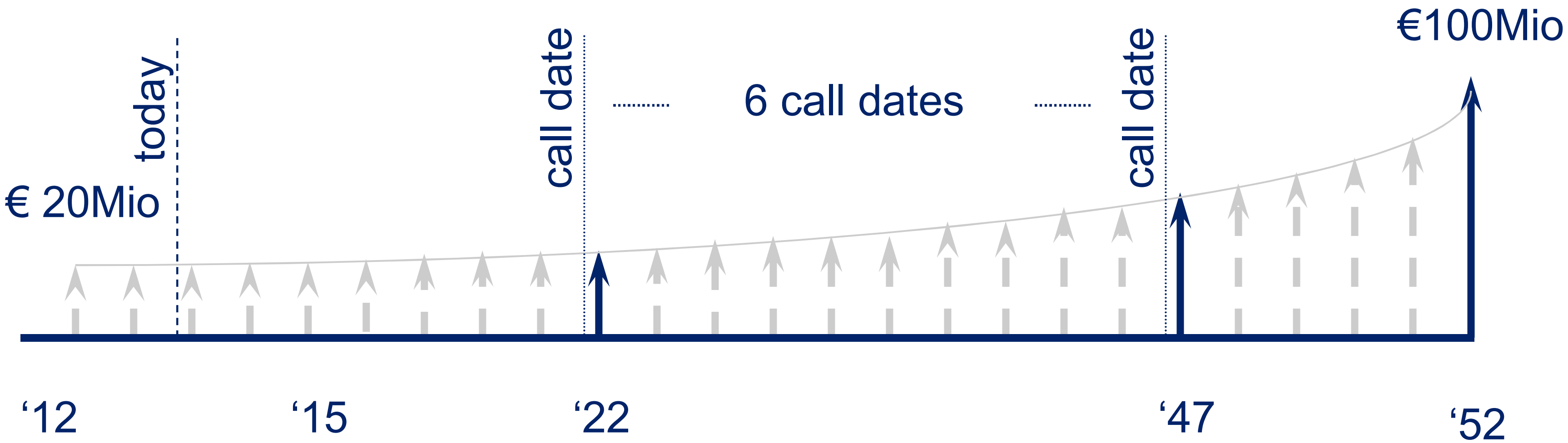
- Observation:
  - Implementation of Swaption-Instrument is tightly bound to the implementation of a VanillaSwap-Instrument 😐
- Suggestion:
  - Need for a Constructor of class VanillaSwap, who allows for all sorts of schedules ☺

Future developments regarding this piece of code:

- automatic calibration of a bermudan swaption (get rid of tedious nested calibration)
- Full fledged LGM model with all sorts of calibrations

Pricing Quality: Example Callable Zerobond

IRR = 4,1055%



1. With original calibration from FO-System:

From FO-System:	Benchmarking:
PV-FO-System: € …..	**PV-QuantLib: € …..**

PV01: € 100k

ΔPV ≈ 0.2 bp

2. Comparison against market prices with own calibration from QuantLib:

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- Main Result: improved fitness of prices to market

- Reason for the observed effects:

  - Because the HK-Prices of Accreting Swaptions are pretty close to the corresponding LGM Prices the new calibration is more consistent than the result of a calibration based on a weighted vol (Black) approach.

~ The End ~

- P.S. Hagan, Methodology for Callable Swaps and Bermudan Exercise into Swaptions
- P.J. Hunt, J.E. Kennedy, Implied interest rate pricing models, Finance Stochast. 2, 275–293 (1998)
- A. Miemiec, QuantLib Code on SourceForge, (2013)