Cross Asset CVA Application

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Quaternion Risk Management

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1 About Quaternion

Specialist risk consulting and solutions, originated 2008

Founders: Bank risk management professionals
Locations: UK, Germany, Ireland
Service: Quantitative analysis, valuation and validation
Specialty: Design and integration of effective solutions based on open source
Systems: Summit, Murex, Kondor+, Kamakura, Quic, Active Pivot, NumeriX, QuantLib
Software: Quaternion Risk Engine (QRE)
Clients: Commercial, state-sponsored and investment banks

Philosophy of turning banking experience into practical solutions
1 Quaternion Product & Offering

**Consulting Services**
Quantitative Analysis for highly structured products
Pricing and Risk System Implementation and Training

**Validation Services**
Independent review of pricing models and their implementations
Valuation of complex asset and derivative portfolios

**Software Services**
Development of point solutions for pricing and risk analysis
Support in-house quantitative development projects

**Software: Quaternion Risk Engine**
Cross Asset CVA Application based on QuantLib
Quaternion Risk Engine (QRE)

Quaternion *RISK ENGINE* is a cross asset CVA application based on QuantLib

Used to benchmark Tier 1 Investment Bank exposure simulation methods for Basel capital calculation and CVA management.
Credit Valuation Adjustment CVA reduces the NPV, counterparty’s default risk.

Debt Valuation Adjustment DVA increases the NPV, own default risk.

\[ NPV = NPV_{\text{collateralised}} - CVA + DVA \]
3 How to compute CVA?

Unilateral CVA “formula”

\[ CVA = \sum LGD \cdot PD \cdot EE \]

Expected exposure

\[ EE = \mathbb{E} \{ [D(t) \ NPV(t)]^+ \} = P(t) \int [NPV(t, x)]^+ \rho(t, x) \, dx \]

European option pricing formula with (semi-) analytical solutions for

- Interest Rate Swaps, Cross Currency Swaps
- FX Forwards, FX Options
- Caps/Floors, Swaptions
- Inflation Swaps

Advantage: Speed and accuracy
How to compute CVA?

Limits of the semi-analytical approach:
• **Netting** – the underlying is in fact a portfolio of transactions
• **Collateral** – compute CVA for collateralised portfolios
• **Structured products** – no analytical option price expression

Generic approach:
• **Monte Carlo** simulation for market scenario generation
• Pricing under scenarios and through time
• NPV cube analysis for EE etc.
2. Quaternion Risk Engine (QRE)

1. **Comprehensive Risk Analytics**
   - CVA/DVA, PFE, VaR/ETL, FVA etc
   - Netting, Collateral, Deal Ageing

2. **Scalable Architecture**
   - Monte Carlo Simulation Framework
   - Cross Asset Evolution Models (IR, FX, INF, EQ, COM, CR)
   - Risk-neutral and real-world measures
   - Parallel Processing, multi-core/CPU

3. **Interfaces and workflow**
   - Browser based user interface for trade capture and application control
   - What-if scenario / pre-trade impact analysis
   - Efficient aggregation through reporting platforms (e.g. **Active Pivot**)

4. **Transparency and Extensibility**
2 Quaternion Risk Engine

Consulting and Execution

**Trade Capture**
- Application Control

**Data Staging**
- Trade Data
- Market Data

- Data Loading XML

**Scenario Generation (Market Evolution)**
- Forward Valuation
- Portfolio Ageing
- Aggregation Netting

**Analytics**
- Positions
- Dates
- Scenarios

**Configured Reports**
- EE
- CVA/DVA
- PFE
- VaR
- CVaR

**Reporting Platforms (e.g. Active Pivot)**
3. **QRE Implementation: Core Application Tasks**

1. Generate paths for
   - Interest rates
   - FX rates
   - Inflation rates (CPI indices and real rates)
   - Credit spreads
   - Commodity prices
   - Equity prices
   Analytical tractability of models helpful to allow large jumps in time to any horizon.

2. Turn simulated “factors” into QuantLib term structures and index fixing history at future times

3. Reprice the portfolio under future market scenarios (~10 bn NPV calls)

4. Aggregation of NPVs across netting sets, collateral accounts, expectations, quantiles (for CVA, FVA, VaR, PFE, …)
The core application needs

- **Limited QuantLib amendments**
- **Various QuantLib extensions** (instruments, models, engines) following QuantLib design and structure, organised as a separate Library
- **Some Wrapper Libraries for “building the forest”**
  - constructing QuantLib/QuantExt objects from external representations (e.g. term structures, portfolios)
  - organising data (market quote and “curves“ repository, etc.)
  - I/O, accessing data (databases, xml files, etc.)
- **Parallel processing** for cube generation in finite time
- **Help in efficient aggregation** of large cubes (~10bn NPVs)
Modules – controlled by scripts and XML files or via Web based front end:

1. **Scenario Generation** – RFE models and market data simulation.
2. **Pricing Library** – Instruments, pricing engines (extended QuantLib)
3. **Cube Generation** – Monte Carlo framework to efficiently assemble the NPV cube, parallel processing (multi-core/CPU)
4. **Cube Analysis** – Aggregation, netting, statistics, report generation
3 QRE: Modules

QRE Front End

User Browser

Network

ActivePivotLive

QRE Engine

QRE Controller

QRE Worker

QRE Analytics

DB Extractor

Libraries

QuantLib

QuantExt

Wrap

Wrap-DB

Rapid XML

JAVA / JSP

JAVASCRIPT

C++

Scripts

XML

Cube Files
Examples:

- SimpleQuote: `setValueSilent()` to bypass observer notification
- SwapIndex: caching of underlying vanilla swaps in a map by fixing date, pass a pricing engine to the constructor
- IborCoupon: Overwrite `amount()` method to avoid coupon pricer
- Some Kronrod integral and Numeric Hagan pricer fixes
- StochasticProcessArray: Expose `SalvagingAlgorithm` to the constructor
- VanillaSwap: Added `fixedAnnuity()` and `floatingAnnuity()` methods
- Swaption: added `impliedNormalVolatility()` method, added `NormalBlackSwaptionEngine`
3 QRE Implementation: QuantLib Extensions

Instruments
- CDO Squared
- Cash Flow CLO
- FX Option Variants
- Amortising Swaption
- CMS Spread Option
- CMS Spread Range Accrual
- Cross Currency Swaption
- Power Reverse Dual Currency Swap
- Equity Basket Option
- Resettable Inflation Swap
- ...

Models
- Linear Gauss Markov (LGM)
- Two-Factor LGM
- Cross/Multi Currency LGM
- Jarrow-Yildirim-LGM (Inflation)
- Dodgson-Kainth-LGM (Inflation)
- Multi-Currency-Inflation
- Black-Karasinski
- Cox-Ingersoll-Ross
- Cox-Ingersoll-Ross with jumps
- Two-Factor Gabillon (Commodity)
- ...

Engines
- Two-Curve Bermudan Swaption with LGMs for Discount and Forward
- Semi-Analytic CDS Option in JCIR
- CPI Cap and YoY Inflation Cap in Jarrow-Yildirim-LGM
- ...

Optimization Methods: ASA, …
3  QRE: Model Extensions for Risk-Neutral Evolution

- **IR/FX:** Multi-Currency Linear Gauss Markov model, calibrated to FX Options, Swaptions, Caps/Floors
- **Inflation:** Jarrow-Yildirim model for CPI and real rate, calibrated to CPI and Year-on-Year Caps/Floors
- **Equity:** Geometric Brownian Motion for the spot prices, deterministic dividend yield, calibrated to Equity Options
- **Commodity:** 2-factor Gabillon model for the futures prices, calibrated to Constant Maturity Commodity indices and futures options
- **Credit:** Cox Ingersoll Ross model with jumps for the hazard rate (SSRJD, JCIR), calibrated to CDS Options
IR, FX, INF, EQ, COM model features:

- Analytically tractable: Terminal expectations and covariances have closed form expressions
- Simulation of arbitrarily large time steps possible
- Quick convergence using low discrepancy sequences
- Fast generation of market scenarios
- Risk-neutral measures: T-Forward, Linear Gauss Markov

Credit (BK, JCIR) numerically more challenging

Similar to Historical Simulation, but more involved to ensure realistic curve shapes over long horizons.

Used for Credit Risk (Potential Future Exposure) and Market Risk measures
Key for overall performance:

- We make extensive use of QuantLib’s observer/observable design: Pricing under a scenario by updating relevant market quotes

- But: Notifying large numbers of observers takes time

- Avoid kicking off observer chains after each quote’s update, rather “silently” update quotes and notify term structures once after all related quotes are updated

- Unregister floating rate coupons with their indices to limit the no. of observers

- Use index and engine factories when building the portfolio (only one instance rather than one per trade) to reduce no. of observers
Key for overall performance:

- We need to rebuild **fixing history** on each path, but adding fixings one by one turned out to be quite slow: Maintain the entire history in memory and call `setHistory()` to copy the entire map to the index manager.

- Build **quicker versions of vanilla engines** where possible. Swap example: Avoid BPS calculation and avoid calling `Cashflows::npv()` which triggers coupon pricers:
  - get pricing time down to ~50 micro seconds
  - impact on swap indices and CMS pricing
GPU experiments

• Speed up selected product’s pricing by rewriting pricing engines in CUDA

• Attainable speed up varies with type of ”problem“:
  Factor 250 (Asian Option) to 10 (bespoke PRDC) using NVIDIA GeForce GT 650M, 384 cores @ 0.9 GHz

• Fine-tuning to target hardware required.

• Limited relevance for the overall portfolio so far
Parallelisation

• Fortunately, bummer #1 is not an obstacle here …

• Multiple processes to generate the NPV cube

• Assigning full portfolio but part of the samples to cores seems perfect for load balancing

• We also assign sub-portfolios to cores each processing all samples; split according to single path “timing run”; advantageous with respect to interfacing into Active Pivot
4  QRE Use Cases

Some Use Cases

• CVA Solution

• Validation and benchmarking of risk factor evolution models used in an IB CVA management and credit exposure system

• Backtesting real-world and risk-neutral risk factor evolution models cross asset classes

• Pricing engine for portfolio backtesting
Thank you
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Appendix
5 QRE – Vanilla Swap Exposure, Uncollateralised

Single Currency Swap, bullet, Q fixed vs. Q floating.

![Graph of Single Currency Swap](image)
QRE – Vanilla Swap Exposure, Uncollateralised

Single Currency Swap, bullet, A fixed vs. Q floating.
5 QRE – Cross Currency Swap, Uncollateralised

Cross Currency Swap, bullet, Q fixed vs. Q floating.

Exposure / EUR vs. Time

E[NPV+] (red) and PFE 90% (blue)
Notional 100m EUR, annual fixed vs 6m Euribor

Threshold 4m EUR, MTA 0.5m EUR, MPR 2 Weeks
QRE – Collateralised Swap, Exposures

Notional 100m EUR, annual fixed vs 6m Euribor
Threshold 4m EUR, MTA 0.5m EUR, MPR 2 Weeks
Notional 100m EUR, annual fixed vs 6m Euribor

Threshold 1m EUR, MTA 0.5m EUR, MPR 2 Weeks
Notional 100m EUR, annual fixed vs 6m Euribor

MPR 2 Weeks
European Swaption Exposure, Expiry 5Y, Cash Settlement
5 QRE – Portfolio Evolution, Cash vs. Physical Settlement

Underlying Swap, Forward Start in 5Y, Term 5Y

![Graph showing comparison between Swaption and Forward Swap over time.](image)
5 QRE – Portfolio Evolution, Cash vs. Physical Settlement

European Swaption with Physical Settlement