Cash Settled Swaption Pricing

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Quaternion Risk Management

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Cash Settled Swaption Arbitrage

How to fix it

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How to fix it

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Market Formula

- Liquid Swaptions for EUR and GBP are cash settled
- Payer Swaption Payoff $C(S)(S-K)^+$ with $C(S)=\sum_{i=1}^{N}\frac{\tau}{(1+\tau S)^i}$
- Market Formula: $P(0,T)C(S_0)\text{Black}(K,S_0,t,\sigma(K))$
- Common knowledge: The market formula is not arbitrage free
- But this was mostly not considered a serious problem and
  - the market formula was used also for ITM options
  - the physical and cash smiles were not distinguished

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A simple arbitrage strategy

- “Zero wide collar” CC = Long payer, short receiver, same strike K
- Matthias Lutz (2015) found a *practical* arbitrage strategy[1]
  - Buy a zero wide collar for some $K > S_0$
  - Hedge this position statically with an ATM zero wide collar
- Hedge Ratio $\Delta = CC_S(K, S_0)/CC_S(S_0, S_0)$
- According to the market formula:
  - Forward Premium $C(S_0)(S_0 - K)$
  - Hedge can be purchased at zero cost
- Payoff: $C(S)(S - K) - \Delta C(S)(S - S_0) - C(S_0)(S_0 - K)$
- This is positive whenever $S \neq S_0$ (and $S > -1/\tau$)

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[1] Two Collars and a Free Lunch, `http://ssrn.com/abstract=2686622`

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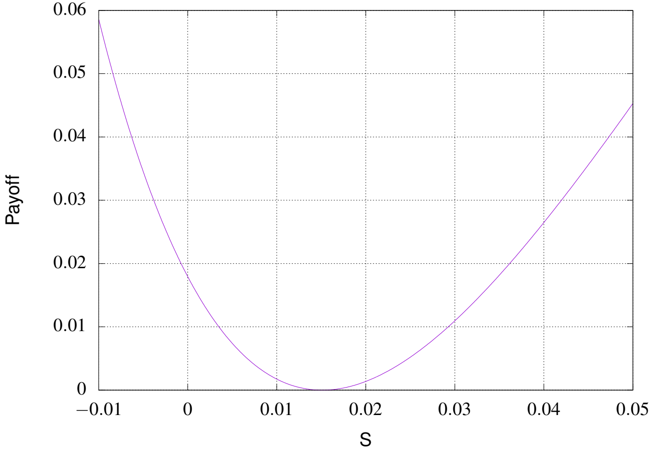
A simple arbitrage strategy

- Payoff for $S_0 = 0.0151$, $K = 0.06$, $N = 30$



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How to fix it

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Vanilla Models

- We need a proper pricing model for Cash Swaptions
- Full Term Structure Models are possible, but heavy
- Instead use a terminal swap rate model model to evaluate

$$A(0)E^{A}\left(\frac{C(t,S)P(t,T)}{A(t,S)}\max(S(t)-K,0)\right)$$

- where
  - t is the fixing and T the settlement time
  - C and A are the cash and physcial annuities respectively

Vanilla Models

- General approach: Specify mapping function

$$M(S(T)) = E^A \left(\frac{P(t,T)}{A(t,S)} \middle| S(t) \right)$$

- M links the underlying swap rate to all discount bonds appearing under the expectation operator
- Once you have that, you can either
  - integrate over the density $\frac{\partial c(t)}{\partial K^2}$ of $S(t)$ implied by the volatility smile
  - use integration by parts to move $\frac{\partial}{\partial K^2}$ from $c(t)$ to the integrand

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Linear TSR

- $M(S(T)) = \alpha S(T) + \beta$
- see `QuantLib::LinearTsrPricer` for such a pricer in the context of CMS coupon pricing
- simple, fast and arbitrage free ...
- ... but for longer maturities possibly unrealistic

Cedervall-Piterbarg Exponential TSR

- Refined TSR approach[2]
- $M(S(T))$ takes into account all relevant swap rates with expiry t, their implied volatilities and correlations
- Stochastic Libor / OIS discounting basis can be incorporated
- Arguably the “state of the art” TSR
- Closer to full term structure models than Linear TSR

[2] Full implications for CMS convexity, Asia Risk, April 2012

Implying the physical smile

- Input is the cash market smile
- From that back out a physical smile, under which the TSR model produces the given market premiums
- For this, choose a parametrisation for the physical smile (e.g. SABR)
- Use a numerical optimisation to fit the physcial smile to the market premiums
- The physical smile is used
  - to price non-quoted cash swaptions (e.g. ITM options)
  - to price physically settled swaptions
  - to calibrate term structure models (since they usually assume a physical input smile)
  - as an input for other vanilla models, e.g. for CMS coupon pricing
- Possibly a simultaneous fit to the cash smile and the CMS market is required

Sample Implementation Steps

- Basis is a TSR Cash Swaption Pricing Engine
- SABR Smile Section that calihbrates to a given grid of input cash volatilities
- With that set up an implied physcial swaption cube
- Possibly, use β to calibrate to CMS, and α, ν, ρ to calibrate to the cash smile

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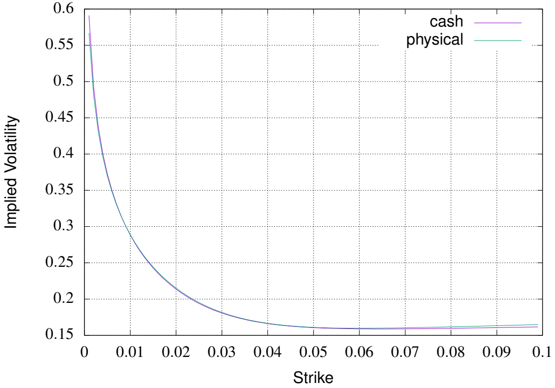
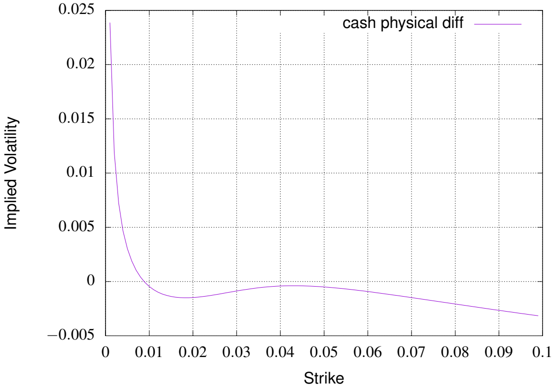
Example Results

- 10Y/10Y, forward 0.03, discount 0.02
- Cash Volatility Input Smile SABR $(0.015, 0.03, 0.2, 0.0)$
- Input cash smile vs. calibrated physical smile (Linear TSR model with one factor reversion 0.05)



Example Results

- Difference cash smile vs. calibrated physical smile:



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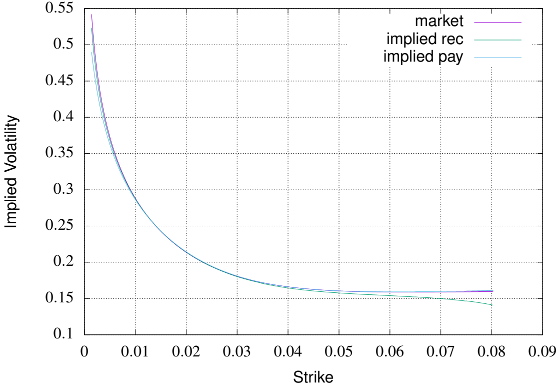
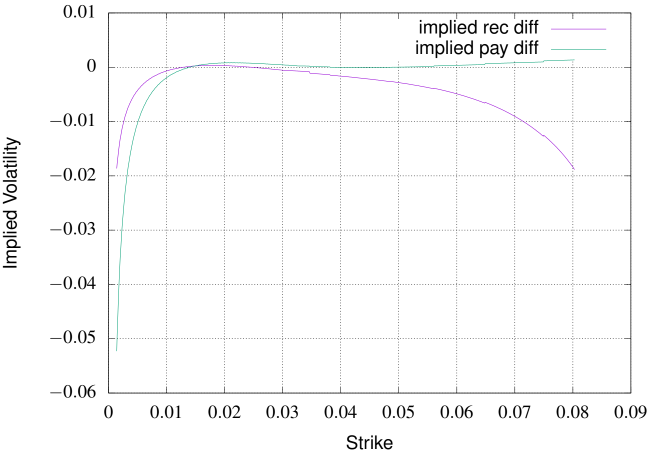
Example Results

- Implied Cash Volatlities after fitting a physcial smile and repricing with Linear TSR model:



Example Results

- Implied Cash Volatlity as Spreads to input volatilities:



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