Solvency II Regulation
How QuantLib can help

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Agenda

- **Solvency II and financial modelling**
- **Building economic scenario generator using QuantLib**
- **Interest rate modelling for Solvency II**
Solvency II
New Regulation for EU Insurance Companies

- In force since 1 January 2016
- Goal is to establish a single regulatory framework for EU insurers and reinsurers
- Inspired by Basel II / III but quite different in details
- Requires market consistent valuation of insurance liabilities
Solvency II
Valuation of Insurance Liabilities

Insurance Portfolio

Legal, Property & Casualty
Focus on actuarial estimation of liability cash flows

Life & Pensions
Main challenge is the valuation of embedded financial options and guarantees

Health Insurance
Combination of actuarial and financial approaches

Valuation approach depends on line of business
Example: Unit linked pension plan (very simplified)

- At inception $t = 0$: minimum guarantee rate $g$ is fixed for pay-out phase
- Accumulation: customer contributions are invested in equity index (without guarantee)
- At retirement $t = T$: the customer savings $S(T)$ are reinvested in risk the free zero bond at interest rate $r(T)$
- At maturity $t = T + M$: customer becomes $S(T) \exp(M \max(g, r(T)))$
- Financial guarantees in this example are equivalent to zero bond option with notional indexed by equity index (option maturity $T$ bond maturity $T + M$).
- Value of financial options and guarantees depends on
  - Equity volatility
  - Rates volatility
  - Correlation
Conventional life and pension insurance policies are much more complicated

- Pay-out of conventional life and pension insurance depends on the performance of investment portfolio.
- Usually a minimum performance rate is guaranteed.
- Some health insurance policies are exposed to inflation risk.
- Value of financial options and guarantees in general depends on volatility and correlations in:
  - Interest rates
  - Equity and property indices
  - Credit spreads
  - Inflation
  - FX
Monte-Carlo simulation is required to determine the value of financial options and guarantees embedded in life, health and pension insurance policies.

Requirements on ESG

- Multi-asset (hybrid) economic model without risk premiums
- Stable projections over very long time horizons of 60-100 years
- Good simultaneous fit to liquid market data
  - Interest rates
  - Interest rate volatility
  - Equity volatility
  - Inflation
  - FX volatility
Economic Scenario Generator

✓ In-house developed in C++ / C# using QuantLib

✓ .NET library which can be used in applications supporting .NET framework
  • In-house developed actuarial and financial applications
  • VBA (e.g. in Excel) and other applications supporting .NET

✓ Configurable via .NET interface or using Excel or Access

✓ Supports calibration, analytical pricing and “on the fly” simulation of hybrid models
  • Interest rates:
    – 1- and 2-Factor Hull-White
    – Cox-Ingersoll-Ross
    – Libor Market Model
  • Equity:
    – Black-Scholes-Merton
    – Heston
  • Inflation
Building Economic Scenario Generator

*Idea:* just put the bricks together

QuantLib offers a big variety of building blocks for financial engineering
Uniform Random Number Generators

- Mersenne Twister: standard RNG with very long period $2^{19937} - 1$
- L'Ecuyer generator
- Knuth’s linear congruential generator

Gaussian Random Number Generators

- Box-Muller transformation
- Inverse cumulative Gaussian

Low Discrepancy Sequences

- Sobol
- Faure
- Halton

Correlation Matrix

- Cholesky decomposition
- Principal Component decomposition
Class **MultiPath** contains list of correlated paths for all assets.

```cpp
MultiPath (Size nAsset, const TimeGrid &timeGrid)
```

Class Template **MultiPathGenerator<GSG>** generates a **MultiPath** from random number generator

**Public Types**

```cpp
typedef Sample< MultiPath > sample_type
```

**Public Member Functions**

```cpp
MultiPathGenerator (const boost::shared_ptr< StochasticProcess > &, const TimeGrid &, GSG generator, bool brownianBridge=false)
```

```cpp
const sample_type & next () const
```

```cpp
const sample_type & antithetic () const
```

*Does not support Brownian bridge (yet)!*
Asset dynamic is defined in a class **StochasticProcess**. This class describes a stochastic process governed by $dx_t = \mu(t, x_t)dt + \sigma(t, x_t)dW_t$. It is the base class for all stochastic models in QuantLib:
Single asset models from QuantLib need to be integrated in a consistent hybrid framework

**Interest rates**
- Hull-White
- Cox-Ingersoll-Ross
- G2
- Libor Market Model

**Equities**
- Black-Scholes-Merton
- Heston
- Bates

**FX**
- Garman-Kolhagen
Libor Market Model was the model of choice at Munich RE Group for the Solvency II preparatory phase (2006 ff.)

*Forward rate dynamic:*

\[
\frac{df_k(t)}{f_k(t)} = \mu(\bar{f}, t)dt + \sigma_k(t)dW_k(t)
\]

**Advantages:**
- Well known in the market
- Good fit to interest rates and ATM swaption volatilities
- Analytical approximations of swaption implied volatilities
- Fast calibration
- No negative rates

**Disadvantages:**
- Rates explosion
- No negative rates
- No volatility skew
Libor Market Model
Coping with Exploding Rates

- Actuarial projection systems became unstable and implausible in case of very high interest rates (>30% – 40%).
- *Naïve capping* of interest rates can produce leakage (violation of martingale property) and significantly distort NAV and risk sensitivity figures.
- **Example**: Investment in cash total return index for t years and reinvestment in 10Y zero coupon bond. This self-financing investment strategy should satisfy martingale property.
Path freezing instead of naïve capping eliminates leakage in actuarial projection models and investment strategies

Idea:
If some forward rate exceeds the capping threshold at some time step in a given scenario, freeze the forwards dynamics from this time step (set the volatility of all forwards to zero)

Why it works:
Stopped martingale is again a martingale. Freezing condition is a stopping time.
Effect of path freezing on model implied volatilities for “reasonable” freezing levels is negligible.
Existence of negative rates can not be ignored anymore.

Idea:
Include displacement in forward rate dynamics
\[
\frac{df_k(t)}{f_k(t) + \delta} = \mu^*(\bar{f}, t)dt + \sigma^*_k(t)dW_k(t)
\]

Advantage:
Existing analytics, user experience and QuantLib implementation can easily be adapted to displaced version
- The drift term \(\mu^*(\bar{f}, t)\) is determined by no-arbitrage arguments.
- Analytical approximations for swaption implied volatilities can be derived following the arguments for non-displaced case:
  - Use “coefficient freezing” approximation to relate model parameters to volatilities in displaced Black-76 model
  - Price swaptions using analytical formulas for displaced Black-76
  - Transform swaption prices to volatility quotes (Black-76 or Normal)
- Parametrization of instantaneous volatility \(\sigma^*_k(t)\) might need to be revisited for displaced case.
Adaptation of the “frozen coefficient” technique leads to a good analytical approximation of swaption implied volatilities

Black-76 model implied volatilities
*Analytical approximation v.s. Monte-Carlo simulation*

![Graph showing Black-76 model implied volatilities for tenors 10 and 20 years.](image)
Some insurance policies are sensitive to interest rate implied volatility skew

- Libor Market Model has (almost) no skew (in Black-76 space)
- Displaced Libor Market Model is not flexible enough to reflect observed market skew

**Market standard:** SABR-Model

\[ d f_k(t) = \sigma_k(t) f_k(t)^{\beta} \, dW_k(t) \]
\[ d\sigma_k(t) = \alpha \sigma_k(t) \, dZ_k(t) \]
\[ dW_k(t) \, dZ_k(t) = \rho \, dt \]
Popular in insurance industry: (Displaced) Wu & Zang Model

Idea: (Displaced) Libor Market Model with Heston-like stochastic volatility

\[
\frac{df_k(t)}{f_k(t) + \delta} = \mu^*(\bar{f}, t) dt + \sqrt{V(t)} \sigma_k^*(t) dW_k(t)
\]
\[
dV(t) = \kappa(\theta - V(t)) dt + \epsilon \sqrt{V(t)} dZ(t)
\]
\[
dW_k(t) dZ(t) = \rho_k dt
\]

Swaption prising:

- “Freezing” the coefficients leads to Heston-like equation for forward swap rates.
- Adapt Heston’s arguments to derive the analytic expressions for moment generating functions for forward rates.
- Swaption prises can be obtained by numerical integration.

• *Stochastic Process*: combine Cox-Ingersoll-Ross for stochastic volatility with displaced Libor Market Model.

• *Semi-analytical Pricing and Calibration*: reuse Heston analytics from QuantLib
Summary

• Pricing of life, pension and health insurance liabilities within Solvency II regulatory framework requires advanced financial models

• QuantLib offers a big variety of ready to use components to build advanced multi asset hybrid models and deal with modelling challenges at insurance companies

• Open source architecture enables fast and efficient adaptation of financial models to insurance specific requirements

• A very time consuming part when using QuantLib for production is documentation