CSA Pricing

Roland Lichters

QuantLib User Meeting, 30 November - 1 December 2015
Agenda

About us

Introduction

CSA Pricing
Agenda

About us

Introduction

CSA Pricing
www.quaternion.com

News:

- Increasing client base in Europe
- Added Quaternion Risk Management US Inc.
- 25 staff and growing in US, IE, UK and DE
- Hybrid offering: Software solutions rooted in quant consulting services for Tier 1 banks
Agenda

About us

Introduction

CSA Pricing
Introduction

Exciting things to do in quant finance

... and using QuantLib
Introduction

Tenor and cross currency basis:
Multi curve pricing, OIS discounting
Negative rates:
Review and revise your pricing models
Introduction

Derivatives catch up with loans:

Valuation Adjustments for Credit, Funding, Capital
Margin requirements for centrally cleared and OTC derivatives:

More VAs - Margin Value Adjustment
Tighter supervision of Internal Model banks:
Credit Exposure simulation for derivatives
Introduction

New standard approach for credit risk capital (SA-CCR):

Impact analysis, comparison to CEM Add-On, and internal model EAD
Optimization of capital and funding cost:

A combination of all of the above
Stress tests and sensitivity analysis on top of xVA and exposures

⇒ It is has got more complicated and more computationally demanding
So there is a need for efficient, clever and transparent tools to cover all this.
Agenda

About us

Introduction

CSA Pricing
CSA Pricing and yet another VA

Ideal CSA

- Symmetric
- Cash collateral in single currency
- Daily margining
 CSA Pricing and yet another VA

Real CSAs are often more complex

- One-sided thresholds
- Optional bond collateral
- Rating triggers
- Collateral currency choice
- Cash collateral compounding rate (Eonia) shifted and floored at zero
New regulations: Increased capital charges for residual risks e.g. due to asymmetric CSAs

Increased cost of risk mitigation measures

Trend to simplify CSAs (ISDA)
CSA Pricing and yet another VA

A real case:

- Harmonize CSAs for adequate compensation, i.e. price the features

- Portfolio: A few thousand Swaps, FX Swaps, Bermudan Swaptions, Inflation Swaps, BMA Swaps, CDS, and a bunch of structured products
What does this involve?

At the high level

- One-sided thresholds
  ⇒ CVA, DVA, FVA

- Bond collateral and rating triggers
  ⇒ Credit modelling

- Collateral currency choice
  ⇒ Cross currency basis modelling

- Cash collateral compounding rate (Eonia)
  shifted and floored at zero ⇒ See next slides
Eonia curves as of 30/09/2014 and 23/10/2015
Methodology

The Eonia floor feature in a CSA has two effects:

1. It affects the fair value of each derivative in the netting set

2. It affects the fair value of future interest cash flows paid/received in the margining process
'Ordinary’ OIS Discounting

\[
\text{Discount}(T) = \mathbb{E} \left[ e^{-\int_0^T r(s) \, ds} \right]
\]

'Floored’ OIS Discounting

\[
\text{Discount}(T) = \mathbb{E} \left[ e^{-\int_0^T r^+(s) \, ds} \right]
\]

does not have closed form, but approximate solutions.
Figure: EONIA forward curve as of 30/09/2014 with negative rates up to 2 years. Under the CSA collateral is paid in EUR and based on EONIA - 10 bp.
Figure: Shifted EONIA forward curve compared to the forward curve with collateral floor; Hull-White parameters are $\lambda = 0.05$ and $\sigma = 0.004$. 
Swap Pricing

Needs more than discounting, floating leg:

$$\Pi_{\text{Float}} = \mathbb{E} \left[ \sum_{i=1}^{n} L(t_{i-1}, t_i) \times \delta(t_{i-1}, t_i) \times D(t_i) \right]$$

$$L(t_{i-1}, t_i) = \frac{1}{\delta(t_{i-1}, t_i)} \left( e^{\int_{t_{i-1}}^{t_i} r(s) \, ds} - 1 \right)$$

$$D(t_i) = e^{-\int_{0}^{t_i} r^+(s) \, ds}$$
It is tempting to build the ‘floored’ discount curve, keep the forward curves unchanged and to do curve pricing as usual.

Unfortunately, this does not yield the ’exact’ Swap value.

Only full MC pricing yields the ’exact’ price, even for a vanilla Swap.
Swap Pricing

The error is noticeable but quite small.

<table>
<thead>
<tr>
<th>Term</th>
<th>no floor</th>
<th>with floor</th>
<th>diff</th>
<th>approx.</th>
<th>error</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-665.21</td>
<td>-663.02</td>
<td>2.19</td>
<td>-663.26</td>
<td>-0.24</td>
</tr>
<tr>
<td>3</td>
<td>-1135.64</td>
<td>-1129.83</td>
<td>5.81</td>
<td>-1130.54</td>
<td>-0.71</td>
</tr>
<tr>
<td>4</td>
<td>-1459.73</td>
<td>-1450.40</td>
<td>9.34</td>
<td>-1451.84</td>
<td>-1.44</td>
</tr>
<tr>
<td>5</td>
<td>-1743.69</td>
<td>-1730.53</td>
<td>13.16</td>
<td>-1732.88</td>
<td>-2.35</td>
</tr>
<tr>
<td>7</td>
<td>-2216.14</td>
<td>-2195.89</td>
<td>20.24</td>
<td>-2200.47</td>
<td>-4.58</td>
</tr>
<tr>
<td>10</td>
<td>-2757.42</td>
<td>-2728.46</td>
<td>28.96</td>
<td>-2735.91</td>
<td>-7.45</td>
</tr>
<tr>
<td>12</td>
<td>-3056.87</td>
<td>-3022.23</td>
<td>34.64</td>
<td>-3031.87</td>
<td>-9.64</td>
</tr>
<tr>
<td>15</td>
<td>-3485.65</td>
<td>-3442.96</td>
<td>42.69</td>
<td>-3455.76</td>
<td>-12.80</td>
</tr>
<tr>
<td>20</td>
<td>-4204.30</td>
<td>-4148.54</td>
<td>55.77</td>
<td>-4165.78</td>
<td>-17.24</td>
</tr>
</tbody>
</table>

Table: Vanilla swaps, 4% fixed vs Euribor 6m flat, Hull White model with $\lambda = 0.01$ and $\sigma = 0.005$, market data as of 30/06/2015. Approx: Only discount curve replaced, forward curve unchanged.
Move on to the second effect:
Impact on interest on collateral
Without Eonia floor, the value of collateral interest cash flows is

\[ \Pi_{\text{NotFloored}} = \mathbb{E} \left[ \sum_i C(t_i) \cdot r(t_i) \cdot \delta_i \cdot D(t_{i+1}) \right] \]

with

- \( C(t_i) \): posted collateral
- \( r(t_i) \): overnight rate applicable to period \((t_i, t_{i+1})\)
- \( \delta_i \): day count fraction for period \((t_i, t_{i+1})\)
- \( D(t_{i+1}) \): stochastic discount factor
Margin Effect, Eonia Floor Value

With floored Eonia rates, the value of collateral interest cash flows is

$$\Pi_{Floored} = \mathbb{E} \left[ \sum_i \widetilde{C}(t_i) \cdot r^+(t_i) \cdot \delta_i \cdot \widetilde{D}(t_{i+1}) \right]$$

where $\widetilde{C}$ and $\widetilde{D}$ denote floor-induced modified collateral amounts and stochastic discount factors.
Margin Effect, Eonia Floor Value, COLVA

In summary

\[ \Pi_{\text{Floor}} = \Pi_{\text{Floored}} - \Pi_{\text{NotFloored}} \]

\[ = \mathbb{E} \left[ \sum_i \left( \tilde{C}(t_i) \tilde{D}(t_{i+1}) \delta_i (r(t_i))^+ - C(t_i) D(t_{i+1}) \delta_i r(t_i) \right) \right] \]

\[ \approx \mathbb{E} \left[ \sum_i C(t_i) D(t_{i+1}) \delta_i (-r(t_i))^+ \right] . \]

So-called **COLVA**, see Burgard-Kjaer, or ...
Methodology

Modern Derivatives Pricing and Credit Exposure Analysis
Theory and Practice of CSA and XVA Pricing, Exposure Simulation and Backtesting
Roland Lichters, Roland Stamm, Donal Gallagher

Hardcover 9781137494832  £60.00 / $95.00

Available from all good booksellers or online at www.palgrave.com

To order in the USA or Canada: T: 888-330-8477
If you are in Australia or New Zealand: E: palgrave@macmillan.com.au
To order in UK or rest of world: T: +44 (0)1256 302866, E: orders@palgrave.com
\[
\Pi_{Floor} \approx \mathbb{E} \left[ \sum_i C(t_i) D(t_{i+1}) \delta_i (-r(t_i))^+ \right]
\]

is the price of a floor

- paying off when overnight rates are negative, i.e. currently in the money
- with stochastic notional given by the amount of posted collateral
- potentially with significant correlation between notional and rate, depending on netting set composition
Implementation

Quantify floor effects by means of

1. Floor-induced single-trade pricing for interest rate, FX and inflation derivatives using bespoke pricing engines

2. Monte Carlo simulation of the netting set collateral in conjunction with simulation of the compounding rate

QuantLib applied in Quaternion Risk Engine
Implementation

Monte Carlo Simulation Framework

- IR: Linear Gauss Markov models
- FX: Geometric Brownian Motion driven by stoch. IR differential
- INF: Jarrow Yildirim
- CR: Cox Ingersoll Ross and Black Karasinski
Example Portfolio as of 23/10/2015

Exposure evolution

Exposure

time / years

© 2015 Quaternion Risk Management Ltd. Roland Lichters
Example Portfolio

Floorlets, intrinsic values

![Graph showing floorlets over time](chart.png)
Example Portfolio

Floorlets

![Floorlet Graph](image)

Floorlet (intrinsic)
Floorlet

© 2015 Quaternion Risk Management Ltd. Roland Lichters
Example Portfolio

Floor, intrinsic

Floor (intrinsic)

floorlet

time / years

© 2015 Quaternion Risk Management Ltd. Roland Lichters
Example Portfolio

Floor

![Graph showing the relationship between time and floor values. The graph includes two lines: one for floor (intrinsic) and another for floor. The x-axis represents time in years (0 to 10), and the y-axis represents floorlet values. The graph shows an upward trend for both lines as time increases.]
CSA ’Eonia Floor’ value

- can be seen as COLVA
- should be computed by full MC simulation due to the correlation between posted collateral and compounding rate
- has significant time value
- is exposed to significant model risk (rate distribution for negative rates)
Questions