



Negative rates in QuantLib

Peter Caspers

Quaternion Risk Management

01 December 2015

Agenda

Negative fixings and implications

QuantLib implementation

Agenda

Negative fixings and implications

QuantLib implementation

History of negative fixings

- ▶ it started with negative EONIA fixings end of 2014
- ▶ then we had negative Euribor 1m fixings, later 3m, even 6m
- ▶ as of 27-Oct-2015 we have a negative CMS2Y fixing (at -3.5 bp)

Implications of negative fixings

- ▶ interest compounding on collateral accounts under
 - ▶ ISDA negative rates protocol (no floor),
 - ▶ DRV (currently still floored @0%, will probably change)
 - ▶ payment reversal in swaps under
 - ▶ ISDA (yes),
 - ▶ DRV (most probably yes)
 - ▶ floored coupons @0% for
 - ▶ bonds
 - ▶ schuldscheindarlehen
 - ▶ loans
- (probably yes, but to be confirmed from the legal side)

Implications on pricing 1/2

- ▶ rate curves should allow for negative forwards
- ▶ lognormal models can not reproduce market prices for zero (or negative strike) floors and break for negative forwards
- ▶ lognormal models can even fail to produce high enough prices for forward levels like $F = +1\%$ or $+2\%$, because e.g. for shifted lognormal models with shift $d \geq 0$, $K \geq -d$

$$c(K)/N(0) \xrightarrow{\sigma \rightarrow \infty} F + d \quad (1)$$

- ▶ you could actually observe this recently by first exploding, then missing implied lognormal volatility quotes for EUR swaptions with long option tenors like 15y+ (“two holes” in the quoted matrix)

Implications on pricing 2/2

- ▶ shifted Black76 and normal Black76 models were established as market models for low and negative rates
- ▶ shifting is generic, e.g. the shifted SABR model has also become part of the new basic standard of market models
- ▶ with a different motivation (produce skew) a shift was introduced in Libor forward models a long time ago
- ▶ new models / model variants are discovered to handle negative rates in a more sophisticated way (free boundary SABR, mixed SABR)
- ▶ other models need adjustments as well (cms replication coupon pricers, Markov functional model)

Agenda

Negative fixings and implications

QuantLib implementation

Negative rates switch

- ▶ `QL_NEGATIVE_RATES`
- ▶ allows for negative zero yields, forwards, increasing discount factors
- ▶

```
+2012-07-31 14:11 Ferdinando Ametrano
+
+ * [r18305] ql/userconfig.hpp, test-suite/piecewiseyieldcurve.cpp:
+
+   defaulted to allow negative rates (define QL_NEGATIVE_RATES) as this
+   is happening for EUR OIS, CHF and German treasury yields, etc.
```

Volatility type

- ▶ `ql/termstructures/volatility/volatilitytype.hpp`
- ▶ distinguishes between normal and (shifted) lognormal volatilities

```
enum VolatilityType { ShiftedLognormal, Normal };
```

Cap floor volatilities

- ▶ market quotes normal or shifted lognormal volatilities, with a constant shift across strikes and tenors

```
OptionletStripper (const boost::shared_ptr<CapFloorTermVolSurface>&,
                  const boost::shared_ptr<IborIndex>& iborIndex_,
                  const Handle<YieldTermStructure>& discount =
                      Handle<YieldTermStructure>(),
                  const VolatilityType type = ShiftedLognormal,
                  const Real displacement = 0.0);
```

Swaption volatilities

- ▶ market quotes normal or shifted lognormal volatilities, with different shifts per underlying
- ▶ swaption cubes inherit the shift structure from their embedded atm matrix
- ▶ swaption volatility cube 1 uses shifted SABR models
- ▶ the shift is bilinearly interpolated in (option, underlying) space

```
SwaptionVolatilityMatrix(  
    const Calendar& calendar,  
    BusinessDayConvention bdc,  
    ...  
    const VolatilityType type = ShiftedLognormal,  
    const std::vector<std::vector<Real> >& shifts  
        = std::vector<std::vector<Real> >());
```

Libor in arrears adjustments

- ▶ convexity adjustment is amended in a straightforward way for shifted lognormal or normal volatilities
- ▶ timing adjustment is generalized at the same time for arbitrary non-natural fixing times¹

```
enum TimingAdjustment { Black76,  
                        BivariateLognormal };  
BlackIborCouponPricer(const Handle<OptionletVolatilityStructure> &v =  
                      Handle<OptionletVolatilityStructure>(),  
                      const TimingAdjustment timingAdjustment = Black76,  
                      const Handle<Quote> correlation =  
                      Handle<Quote>(boost::make_shared<SimpleQuote>(1.0)))
```

¹see <http://ssrn.com/abstract=2170721>

Linear TSR pricer for cms coupons

- ▶ volatility type is recognized through the abstraction of `SmileSection`
- ▶ the replication range is shifted appropriately (e.g. user bounds set to $[0, 200\%]$ are transformed to $[-1\%, 199\%]$ automatically if the applicable shift is 1% to keep the user input universal under changing shifts in market quotations)
- ▶ for a normal model, the replication domain extends to $(-\infty, \infty)$

CMS spread option pricer

- ▶ swap rate adjustments use shifted lognormal or normal smiles to determine the drifts of the single swap rate models
- ▶ the bivariate model for the swap rates is still purely lognormal currently, which works technically as long as the underlying forward levels are still positive
- ▶ with negative 2Y fixings, we will need to extend this pricer as well!
- ▶ PR #264 allow for shifts in the single rate models or for normal single rate models²

²see <http://ssrn.com/abstract=2686998>

Calibration helpers

- ▶ can be set up with normal and shifted lognormal volatilities
- ▶ cooperative with HullWhite, Gsr, Lgm, MarkovFunctional models

```
SwaptionHelper(const Period& maturity,  
               const Period& length,  
               const Handle<Quote>& volatility,  
               ...  
               const VolatilityType type = ShiftedLognormal,  
               const Real shift = 0.0);
```


Markov functional model

- ▶ replicates a market smile / density per expiry via the numeraire calibration
- ▶ therefore also replicates the density for negative strike ranges
- ▶ currently, only shifted lognormal smile input allowed
- ▶ todo: allow normal smile input for numeraire calibration

UK

29th Floor, 1 Canada Square
Canary Wharf, London E145DY
+44 207 712 1645
caroline.tonkin@quaternionrisk.com

Germany

Maurenbrecherstrasse 16
47803 Krefeld
+49 2151 9284 800
heidy.koenings@quaternionrisk.com

Ireland

54 Fitzwilliam Square
Dublin 2
+353 1 678 7922
joelle.higgins@quaternionrisk.com

UK



Germany



Ireland

